

# Differenzialrechnung

## Aufgabenblatt Ableitungen

### zur Produkt- und Quotientenregel

Lösungen

Level 1 – Grundlagen – Blatt 2

#### Lösung A1

$f_1(x) = (3x - 2) \cdot (x^2 - 1)$	$f_1'(x) = x \cdot (x^2 - 1) + 2x \cdot (3x - 2)$
$f_2(x) = (3 - x^2) \cdot (x^2 - 2)$	$f_2'(x) = -2x \cdot (x^2 - 2) + 2x \cdot (3 - x^2)$
$f_3(x) = (x - 1)^2 \cdot (x + 1)^2$	$f_3'(x) = 2(x - 1) \cdot (x + 1) + 2(x + 1) \cdot (x - 1)^2$
$f_4(x) = (2x^3 - 2x) \cdot \frac{1}{x}$	$f_4'(x) = (6x - 2) \cdot \frac{1}{x} - \frac{1}{x^2} \cdot (2x^3 - 2x)$
$f_5(x) = (5x^2 - 3) \cdot (-x^2)$	$f_5'(x) = 10x \cdot (-x^2) - 2x \cdot (5x^2 - 3)$
$f_6(x) = (x^2 - 3x) \cdot 4x^3$	$f_6'(x) = (2x - 3) \cdot 4x^3 + 12x^2 \cdot (x^2 - 3x)$
$f_7(x) = (x - 20)^5 \cdot (x + 10)^{-2}$	$f_7'(x) = 5(x - 20)^4 \cdot (x + 10)^{-2} - 2(x + 10)^{-3} \cdot (x - 20)^5$
$f_8(x) = (x^2 - x + 2) \cdot \sin(t)$	$f_8'(x) = 2x - 1$
$f_9(x) = (3x^3 + 4) \cdot \frac{1}{x^2}$	$f_9'(x) = 9x^2 \cdot \frac{1}{x^2} - 2(3x^3 + 4) \cdot \frac{1}{x^3} = 9 - 6 - \frac{4}{x^3} = 3 - \frac{8}{x^3}$

#### Lösung A2

$f_1'(x) = \frac{1}{4}(x - 1)^{-\frac{1}{2}} \cdot (x + 1)^{\frac{1}{2}} + \frac{1}{4}(x + 1)^{-\frac{1}{2}} \cdot (x - 1)^{\frac{1}{2}}$	$f_{18}(x) = 0,5 \cdot (x - 1)^{\frac{1}{2}} \cdot (x + 1)^{\frac{1}{2}}$
$f_2'(x) = \cos^2(x) - \sin^2(x)$	$f_{17}(x) = \sin(x) \cdot \cos(x)$
$f_3'(x) = -\pi \sin(x - 3) \cdot (x^2 - 2) + 2\pi x \cdot \cos(x - 3)$	$f_{16}(x) = \pi \cos(x - 3) \cdot (x^2 - 2)$
$f_4'(x) = 3 \cdot (4 - 3x)^{-2} + 3 \cdot (4 - 3x)^{-1}$	$f_{15}(x) = (4 - 3x)^{-1} \cdot (3x - 4)$
$f_5'(x) = -5x^4 \cdot (x + 3)^2 - 2x^5 \cdot (x + 3)$	$f_{14}(x) = -x^5 \cdot (x + 3)^2$
$f_6'(x) = (20x^3 - 12x^2) \cdot \frac{a}{x^2} - 2(5x^4 - 4x^3) \cdot \frac{a}{x^3}$	$f_{13}(x) = (5x^4 - 4x^3) \cdot \frac{a}{x^2}$
$f_7'(x) = (15x^2 + 2x) \cdot \frac{1}{x} - (5x^3 + x^2 - 4) \cdot \frac{1}{x^2}$	$f_{12}(x) = (5x^3 + x^2 - 4) \cdot \frac{1}{x}$
$f_8'(x) = (2x - 2) \cdot \sin(x + 2) + (x^2 - 2x) \cdot \cos(x + 2)$	$f_{11}(x) = (x^2 - 2x) \cdot \sin(x + 2)$
$f_9'(x) = (8x - 3) \cdot x + (4x^2 - 3x)$	$f_{10}(x) = (4x^2 - 3x) \cdot x$

#### Lösung A3

- a)  $f(x) = 2x^3 \cdot 4x^2 \quad u = 2x^3 \quad u' = 6x^2 \quad v = 4x^2 \quad v' = 8x$   
 $f'(x) = 6x^2 \cdot 4x^2 + 2x^3 \cdot 8x = 24x^4 + 16x^4 = 30x^4$   
 $f''(x) = 120x^3$
- b)  $f(x) = x^3 \cdot (2x^2 - 4) \quad u = x^3 \quad u' = 3x^2 \quad v = 2x^2 - 4 \quad v' = 4x$   
 $f'(x) = 3x^2 \cdot 2x^2 + x^3 \cdot 4x = 6x^4 + 4x^4 = 10x^4$   
 $f''(x) = 40x^3$
- c)  $f(x) = 2(x^3 - x)(x^2 + x) \quad u = x^3 - x \quad u' = 3x^2 - 1 \quad v = x^2 + x \quad v' = 2x + 1$   
 $f'(x) = 2((3x^2 - 1) \cdot (x^2 + x) + (x^3 - x) \cdot (2x + 1))$   
 $= 2(3x^4 + 3x^3 - x^2 - x + 2x^4 + x^3 - 2x^2 - x)$   
 $= 2(5x^4 + 4x^3 - 3x^2 - 2x)$   
 $f''(x) = 2(20x^3 + 12x^2 - 6x - 2)$

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- d)  $f(x) = (x - 1)(x - k)^2 \quad u = x - 1 \quad u' = 1 \quad v = (x - k)^2 \quad v' = 2(x - k)$   
 $f'(x) = (x - k)^2 + 2(x - 1) \cdot (x - k)$   
 $= (x - k) \cdot (x - k + 2x - 2)$   
 $= (x - k) \cdot (3x - k - 2)$   
 $f''(x) = (3x - k - 2) + 3(x - k) = 3x - k - 2 + 3x - 3k$   
 $= 6x - 4k - 2$
- e)  $f(x) = 2ax(x - a)^2 \quad u = 2ax \quad u' = 2a \quad v = (x - a)^2 \quad v' = 2(x - a)$   
 $f'(x) = 2a \cdot (x - a)^2 + 4ax \cdot (x - a)$   
 $= 2ax^2 - 4a^2x + 2a^3 + 4ax^2 - 4a^2x$   
 $= 6ax^2 - 8a^2x + 2a^3$   
 $f''(x) = 12ax - 8a^2$
- f)  $f(x) = 2x \cdot (x^2 + 2) \quad u = 2x \quad u' = 2 \quad v = x^2 + 2 \quad v' = 2x$   
 $f'(x) = (x^2 + 2) + 2x \cdot 2x = x^2 + 2 + 4x^2 = 5x^2 + 2$   
 $f''(x) = 10x$