

Aufgabenblatt Ableitungen

zur Produkt- und Quotientenregel

Lösungen

Level 2 – Fortgeschritten – Blatt 1

Lösung A1

a) $f(x) = \sqrt{x} \cdot (x - 1)^4$ $u = \sqrt{x}$ $u' = \frac{1}{2\sqrt{x}}$ $v = (x - 1)^4$ $v' = 4(x - 1)^3$

$$f'(x) = \frac{1}{2\sqrt{x}} \cdot (x - 1)^4 + 4\sqrt{x} \cdot (x - 1)^3 = \frac{(x-1)^4+8x(x-1)^3}{2\sqrt{x}} = \frac{(x-1)^3(9x-1)}{2\sqrt{x}}$$

$$= \frac{1}{2}(x - 1)^3 \cdot (9x - 1) \cdot x^{-\frac{1}{2}} \quad u = \frac{1}{2}(x - 1)^3 \quad u' = \frac{3}{2}(x - 1)^2$$

$$v = 9x^{\frac{1}{2}} - x^{-\frac{1}{2}} \quad v' = \frac{9}{4\sqrt{x}} + \frac{1}{4x\sqrt{x}}$$

$$f''(x) = \frac{3}{2}(x - 1)^2 \cdot \frac{(9x-1)}{\sqrt{x}} + \frac{1}{2}(x - 1)^3 \cdot \left(\frac{9x+1}{2x\sqrt{x}}\right) = \frac{1}{2\sqrt{x}}(x - 1)^2 \cdot \left(27x - 3 + (x - 1) \cdot \frac{9x+1}{2x}\right)$$

$$= \frac{1}{2\sqrt{x}}(x - 1)^2 \cdot \left(\frac{(27x-3) \cdot 2x + (x-1) \cdot (9x+1)}{2x}\right) = \frac{(x-1)^2 \cdot (63x^2 - 14x - 1)}{4x\sqrt{x}}$$

b) $f(x) = x^2 \cdot (1 - x)^5$ $u = x^2$ $u' = 2x$
 $v = (1 - x)^5$ $v' = -5(1 - x)^4$

$$f'(x) = 2x \cdot (1 - x)^5 - 5x^2 \cdot (1 - x)^4 = (1 - x^4) \cdot (2x \cdot (1 - x) - 5x^2)$$

$$= (1 - x)^4 \cdot (-7x^2 + 2x) \quad u = (x - 1)^4 \quad u' = 4(x - 1)^3$$

$$v = -7x^2 + 2x \quad v' = -14x + 2$$

$$f''(x) = 4(x - 1)^3 \cdot (-7x^2 + 2x) + (x - 1)^4 \cdot (-14x + 2) =$$

$$= 2(x - 1)^3 \cdot (2(-7x^2 + 2x) + (x - 1) \cdot (-7x + 1)) =$$

$$= 2(x - 1)^3 \cdot (-14x^2 + 4x - 7x^2 + x + 7x - 1) = 2(x - 1)^3 \cdot (-21x^2 + 12x - 1)$$

c) $f(x) = \sqrt{x} \cdot (5 + \sqrt{x}) = 5\sqrt{x} + x$

$$f'(x) = \frac{5}{2\sqrt{x}} + 1$$

$$f''(x) = -\frac{5}{4x\sqrt{x}}$$

d) $f(x) = (3x^5 - 2x) \cdot \frac{2}{x}$ $u = 3x^5 - 2x$ $u' = 15x^4 - 2$

$$v = \frac{2}{x} \quad v' = -\frac{2}{x^2}$$

$$f'(x) = (15x^4 - 2) \cdot \frac{2}{x} - (3x^5 - 2x) \cdot \frac{2}{x^2} = \frac{2}{x^2} \cdot \left(x \cdot (15x^4 - 2) - (3x^5 - 2x)\right)$$

$$= \frac{2}{x^2} \cdot (12x^5) = 24x^3$$

$$f''(x) = 72x^2$$

e) $f(x) = \sqrt{x} \cdot \left(5x + \frac{1}{x}\right) = 5x^{\frac{3}{2}} + \frac{1}{x^{\frac{1}{2}}}$

$$f'(x) = \frac{15}{2}\sqrt{x} - \frac{1}{x\sqrt{x}}$$

$$f''(x) = \frac{15}{4\sqrt{x}} + \frac{3}{2x^2\sqrt{x}}$$

f) $f(x) = (5\sqrt{x} + 3x) \cdot 5x = 25x^{\frac{3}{2}} + 15x^2$

$$f'(x) = \frac{75}{2}\sqrt{x} + 30x$$

$$f''(x) = \frac{75}{4\sqrt{x}} + 30$$

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Lösung A2

- a) $f(x) = \frac{2}{3}x^3 \cdot (5x^2 - x + 7)$
- $$u = \frac{2}{3}x^3 \quad u' = 2x^2$$
- $$v = 5x^2 - x + 7 \quad v' = 10x - 1$$
- $$f'(x) = 2x^2 \cdot (5x^2 - x + 7) + \frac{2}{3}x^3 \cdot (10x - 1) = 10x^4 - 2x^3 + 14x^2 + \frac{20}{3}x^4 - \frac{2}{3}x^3$$
- $$= \frac{50}{3}x^4 - \frac{8}{3}x^3 + 14x^2$$
- $$f''(x) = \frac{200}{3}x^3 - 8x^2 + 28x$$
-
- b) $f(x) = \left(\frac{2}{x^2} - 1\right) \cdot (x + 5)$
- $$u = \frac{2}{x^2} - 1 \quad u' = -\frac{4}{x^3}$$
- $$v = x + 5 \quad v' = 1$$
- $$f'(x) = -\frac{4}{x^3} \cdot (x + 5) + \frac{2}{x^2} - 1 = -\frac{4}{x^2} - \frac{20}{x^3} + \frac{2}{x^2} - 1 = -\frac{20}{x^3} - \frac{2}{x^2} - 1$$
- $$f''(x) = \frac{60}{x^4} + \frac{4}{x^3}$$
-
- c) $f(x) = (5\sqrt{x} + 3x) \cdot (5x + 7)$
- $$u = 5\sqrt{x} + 3x \quad u' = \frac{5}{2\sqrt{x}} + 3$$
- $$v = 5x + 7 \quad v' = 5$$
- $$f'(x) = \left(\frac{5}{2\sqrt{x}} + 3\right)(5x + 7) + 5 \cdot (5\sqrt{x} + 3x) = \frac{25\sqrt{x}}{2} + \frac{35}{2\sqrt{x}} + 15x + 21$$
- $$f''(x) = \frac{25}{4\sqrt{x}} - \frac{35}{4x\sqrt{x}} + 15 = \frac{25x - 35}{4x\sqrt{x}} + 15$$
-
- d) $f(x) = \left(-\frac{1}{x^2} + \frac{1}{10}x\right) \cdot \sqrt{x}$
- $$u = -\frac{1}{x^2} + \frac{1}{10}x \quad u' = \frac{2}{x^3} + \frac{1}{10}$$
- $$v = \sqrt{x} \quad v' = \frac{1}{2\sqrt{x}}$$
- $$f'(x) = \left(\frac{2}{x^3} + \frac{1}{10}\right) \cdot \sqrt{x} + \frac{1}{2\sqrt{x}} \cdot \left(-\frac{1}{x^2} + \frac{1}{10}x\right) = \frac{2}{x^2\sqrt{x}} + \frac{1}{10}\sqrt{x} - \frac{1}{2x^2\sqrt{x}} + \frac{1}{20}\sqrt{x}$$
- $$= \frac{3}{2x^2\sqrt{x}} + \frac{3}{20}\sqrt{x}$$
- $$f''(x) = -\frac{15}{4x^3\sqrt{x}} + \frac{3}{40\sqrt{x}}$$
-
- e) $f(x) = \sqrt{x} \cdot \left(5x + \frac{1}{x}\right)$
- $$u = \sqrt{x} \quad u' = \frac{1}{2\sqrt{x}}$$
- $$v = 5x + \frac{1}{x} \quad v' = 5 - \frac{1}{x^2}$$
- $$f'(x) = \frac{1}{2\sqrt{x}} \cdot \left(5x + \frac{1}{x}\right) + \sqrt{x} \cdot \left(5 - \frac{1}{x^2}\right) = \frac{5}{2}\sqrt{x} + \frac{1}{2x\sqrt{x}} + 5\sqrt{x} - \frac{1}{x\sqrt{x}}$$
- $$= \frac{15}{2}\sqrt{x} - \frac{1}{2x\sqrt{x}}$$
- $$f''(x) = \frac{15}{4\sqrt{x}} + \frac{3}{4x^2\sqrt{x}}$$
-
- f) $f(x) = (5\sqrt{x} + 3x) \cdot 5x$
- $$u = 5\sqrt{x} + 3x \quad u' = \frac{5}{2\sqrt{x}} + 3$$
- $$v = 5x \quad v' = 5$$
- $$f'(x) = \frac{25}{2\sqrt{x}} + 5 \cdot (5\sqrt{x} + 3x) = \frac{25}{2}\sqrt{x} + 25\sqrt{x} + 15x$$
- $$= \frac{75}{2}\sqrt{x} + 15x$$
- $$f''(x) = \frac{75}{4\sqrt{x}} + 15$$

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Lösung A3

a) $f_1(x) = \left(\frac{1}{x} + 1\right) \cdot (3x + 5)$

$$u = \frac{1}{x} + 1 \quad u' = -\frac{1}{x^2}$$

$$v = 3x + 5 \quad v' = 3$$

$$f_1'(x) = -\frac{3x+5}{x^2} + 3\left(\frac{1}{x} + 1\right)$$

$$f_1'(-1) = -2$$

b) $f_2(x) = 2x^6 \cdot \left(\frac{1}{x^3} - \frac{1}{x^4}\right)$

$$u = 2x^6 \quad u' = 12x^5$$

$$v = \frac{1}{x^3} - \frac{1}{x^4} \quad v' = -\frac{3}{x^4} + \frac{4}{x^5}$$

$$f_2'(x) = 12x^5\left(\frac{1}{x^3} - \frac{1}{x^4}\right) + 2x^6\left(-\frac{3}{x^4} + \frac{4}{x^5}\right)$$

$$f_2'(2) = 24 - 8 = 16$$

c) $f_3(x) = \sqrt{x} \cdot \left(5x^2 + \frac{1}{x^2}\right)$

$$u = \sqrt{x} \quad u' = \frac{1}{2\sqrt{x}}$$

$$v = 5x^2 + \frac{1}{x^2} \quad v' = 10x - \frac{2}{x^3}$$

$$f_3'(x) = \frac{1}{2\sqrt{x}}\left(5x^2 + \frac{1}{x^2}\right) + \sqrt{x}\left(10x - \frac{2}{x^3}\right)$$

$$f_3'(1) = 3 + 8 = 11$$

d) $f_4(x) = \left(\frac{1}{8}x^3 + \frac{6}{5}x\right) \cdot \left(\frac{2}{5}x - 8\right)$

$$u = \frac{1}{8}x^3 + \frac{6}{5}x \quad u' = \frac{3}{8}x^2 + \frac{6}{5}$$

$$v = \frac{2}{5}x - 8 \quad v' = \frac{2}{5}$$

$$f_4'(x) = \left(\frac{3}{8}x^2 + \frac{6}{5}\right)\left(\frac{2}{5}x - 8\right) + \frac{2}{5}\left(\frac{1}{8}x^3 + \frac{6}{5}x\right) \quad f_4'(0) = -\frac{48}{5}$$

Lösung A4

a) $f(x) = (x + 3x^2) \cdot (2x - 1)$

$$u = x + 3x^2 \quad u' = 1 + 6x$$

$$v = 2x - 1 \quad v' = 2$$

$$f'(x) = (1 + 6x) \cdot (2x - 1) + 2(x + 3x^2) = 12x^2 - 2x - 1$$

$$g(x) = 6x^3$$

$$g'(x) = 12x^2$$

$$f'(x) \cap g'(x)$$

$$12x^2 - 2x - 1 = 12x^2$$

$$-2x - 1 = 0$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

$$f'\left(-\frac{1}{2}\right) = 3 \quad g'\left(-\frac{1}{2}\right) = 3$$

An der Stelle $x_0 = -\frac{1}{2}$ verlaufen die beiden Graphen mit einer Steigung von $m = 3$ parallel.

b) $f(x) = \left(\frac{1}{2}x^2 + 8x\right) \cdot \frac{1}{x}$

$$u = \frac{1}{2}x^2 + 8x \quad u' = x + 8$$

$$v = \frac{1}{x} \quad v' = -\frac{1}{x^2}$$

$$f'(x) = (x + 8) \cdot \frac{1}{x} - \frac{1}{x^2}\left(\frac{1}{2}x^2 + 8x\right) = 1 + \frac{8}{x} - \frac{1}{2} - \frac{8}{x} = \frac{1}{2}$$

$$g(x) = -\frac{1}{x^3} + 3$$

$$g'(x) = \frac{3}{x^4}$$

$$f'(x) \cap g'(x)$$

$$\frac{1}{2} = \frac{3}{x^4}$$

$$x_{1,2} = \pm\sqrt[4]{6}$$

An den Stellen $x_1 = \sqrt[4]{6}$ und $x_2 = -\sqrt[4]{6}$ verlaufen die beiden Graphen mit einer Steigung von $m = \frac{1}{2}$ parallel.