Lösung A1

Differenzenquotient:

$$\frac{\Delta y}{\Delta x} = \frac{g(x+h)-g(x)+k(x+h)-k(x)}{x+h-x} = \frac{3(x+h)^2-3x^2+(x+h+2)^3-(x+2)^3}{h}$$

Daraus folgt:

$$\frac{\Delta y}{\Delta x} = \frac{3x^2+6hx+3h^2-3x^2+(x+h+2)^2\cdot(x+h+2)-(x^3+6x^2+12x+8)}{h}$$

$$= \frac{6hx+3h^2+(x^2+2hx+4x+4h+h^2+4)(x+h+2)-x^3-6x^2-12x-8}{h}$$

$$= \frac{h(6x+3h)+x^3+3hx^2+6x^2+3h^2x+12hx+12x+h^3+6h^2+12h+8-x^3-6x^2-12x-8}{h}$$

$$= \frac{h(6x+3h)+3hx^2+3h^2x+12hx+12x+h^3+6h^2+12h}{h}$$

$$= \frac{\cancel{h}(6x+3h)+\cancel{h}(3x^2+3hx+12x+h^2+6h+12)}{\cancel{h}}$$

$$= (6x+3h)+(3x^2+3hx+12x+h^2+6h+12)$$

$$\frac{dy}{dx} = f'(x) = \lim_{h\to 0}(6x+3h)+(3x^2+3hx+12x+h^2+6h+12) = (6x)+(3x^2+12x+12)$$

$g(x) = 3x^2;$ $\qquad g'(x) = 6x$

$k(x) = (x+2)^3 = x^3+6x^2+12x+8$ $\qquad k'(x) = 3x^2+12x+12$

$f'(x) = g'(x)+k'(x) = 6x+3x^2+12x+12$

q.e.d.