

Aufgabenblatt Ableitungen

vermischte Aufgaben

Differenzialrechnung

Lösungen

Level 2 – Fortgeschritten – Blatt 2

Lösung A1

- a) $f'(x) = e^x + 2x \quad f''(x) = e^x + 2$
 b) $f'(x) = 3e^x - x + 1 \quad f''(x) = 3e^x - 1$
 c) $f'(x) = -e^{-x} + e^x \quad f''(x) = e^{-x} + e^x$
 d) $f'(x) = -2e^{-2x} + 4e^{-x} = 2e^{-x}(2 - e^{-x}) \quad f''(x) = 4e^{-2x} - 4e^{-x} = 4e^{-x}(e^{-x} - 1)$
 e) $f'(x) = (2x - 2) \cdot e^x + (x^2 - 2x - 1) \cdot e^x$
 $= e^x(2x - 2 + x^2 - 2x - 1) = e^x(x^2 - 3) \quad f''(x) = e^x(x^2 + 2x - 3)$
 f) $f'(x) = e^x \cdot (3x - 1) \quad f''(x) = e^x(3x + 2)$
 g) $f'(x) = 6x \cdot e^{-4x} - 12x^2 \cdot e^{-4x} = 6e^{-4x}(x - 2x^2) \quad f''(x) = e^{-4x}(48x^2 - 48x + 6)$
 h) $f'(x) = \frac{3}{2}x^2 \cdot e^{2x} + x^3 \cdot e^{2x} = e^{2x}\left(x^3 + \frac{3}{2}x^2\right) \quad f''(x) = e^{2x}(2x^3 + 6x^2 + 3x)$
 i) $f'(x) = -e^{-x}(2x + 3) \quad f''(x) = e^{-x}(2x + 1)$
 j) $f'(x) = e^{-kx}(1 - k(x + k)) = -e^{-kx}(kx + k^2 - 1) \quad f''(x) = ke^{-kx}(kx + k^2 - 2)$
 k) $f'(x) = 2(4x + e^{-x})(4 - e^{-x}) \quad f''(x) = e^{-2x}(32e^{2x} + e^x(8x - 16) + 4)$
 l) $f'(x) = 2e^{-2x} \cdot (e^{4x} - 1) \quad f''(x) = 4e^{-2x}(e^{4x} + 1)$
 m) $f'(x) = e^{2x+1} \cdot (2x + 7) \quad f''(x) = 4e^{2x+1}(x + 4)$
 n) $f'(x) = e^{-0,5x} \cdot (2x - 8) \quad f''(x) = -e^{-0,5x} \cdot (x - 6)$
 o) $f'(x) = -e^{1-x} \cdot (x^2 - 2) \quad f''(x) = e^{1-x} \cdot (x^2 - 2x - 2)$
 p) $f_a'(x) = -\frac{x+2a-1}{e^x} \quad f_a''(x) = \frac{x+2a-2}{e^x}$
 q) $f'(x) = -e^{-0,6x}(48e^{0,12x} - 60) \quad f''(x) = e^{-1,08x}(23,04e^{0,6x} - 36e^{0,48x})$
 r) $f_a'(x) = 2e^x(e^x - a) \quad f_a''(x) = 2e^x(2e^x - a)$
 s) $N_k'(t) = -kN_0 \cdot e^{-2kt}(e^{kt} - 2) \quad N_k''(t) = k^2N_0e^{-2kt}(e^{kt} - 4)$
 t) $f_a'(x) = -a^2x \cdot e^{1-ax} \quad f_a''(x) = a^2e^{1-ax}(ax - 1)$
 u) $f_a'(t) = \frac{2ae^x}{(e^x+a)^2} \quad f_a''(t) = -\frac{2ae^x(e^x-a)}{(e^x+a)^3}$
 v) $f_t(x) = \frac{e^{tx}-e^{-tx}}{e^{tx}+e^{-tx}}$
 $u = e^{tx} - e^{-tx} \quad u' = te^{tx} + te^{-tx}$
 $v = e^{tx} + e^{-tx} \quad v' = te^{tx} - te^{-tx}$
 $f_t(x)' = \frac{u'v-v'u}{v^2} = \frac{(te^{tx}+te^{-tx}) \cdot (e^{tx}+e^{-tx}) - (te^{tx}-te^{-tx}) \cdot (e^{tx}-e^{-tx})}{(e^{tx}+e^{-tx})^2}$
 $f_t(x)' = \frac{te^{2tx}+t+t+te^{-2tx}-(te^{2tx}-t-t+e^{-2tx})}{(e^{tx}+e^{-tx})^2}$
 $f_t(x)' = \frac{4t}{(e^{tx}+e^{-tx})^2} \quad | \quad \text{Erweitern mit } e^{2tx}$
 $f_t(x)' = \frac{4te^{2tx}}{e^{2tx} \cdot (e^{tx}+e^{-tx})^2} = \frac{4te^{2tx}}{e^{4tx}+2e^{2tx}+1}$
 $f_t(x)' = \frac{4te^{2tx}}{(e^{2tx}+1)^2}$
 $u = 4te^{2tx} \quad u' = 8t^2e^{2tx}$
 $v = (e^{2tx} + 1)^2 \quad v' = 4te^{2tx}(e^{2tx} + 1)$
 $f_t(x)'' = \frac{8t^2e^{2tx} \cdot (e^{2tx}+1)^2 - 16t^2e^{4tx}(e^{2tx}+1)}{(e^{2tx}+1)^4}$
 $f_t(x)'' = \frac{8t^2e^{2tx} \cdot (e^{2tx}+1) - 16t^2e^{4tx}}{(e^{2tx}+1)^3} = \frac{8t^2e^{4tx}+8t^2e^{2tx}-16t^2e^{4tx}}{(e^{2tx}+1)^3}$
 $f_t(x)'' = \frac{8t^2e^{2tx}-8t^2e^{4tx}}{(e^{2tx}+1)^3} = \frac{8t^2e^{2tx}(1-e^{2tx})}{(e^{2tx}+1)^3}$

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w) $f'(x) = \frac{(4x-3)e^{2x}}{2x^2\sqrt{x}}$

$$f''(x) = \frac{(16x^2 - 24x + 15)e^{2x}}{4x^3\sqrt{x}}$$

Lösung A2

$$f(x) = 2x \cdot e^{-x}$$

$$f'(x) = -e^{-x}(2x - 2)$$

$$f''(x) = e^{-x}(2x - 4)$$

$$f'''(x) = -e^{-x}(2x - 6)$$

Die n -te Ableitung errechnet sich über die Formel:

$$f^{(n)}(x) = (-1)^n \cdot e^{-x} \cdot (2x - 2n); \quad n \in \mathbb{N}^*$$

Somit ist $f^{(10)}(x) = e^{-x}(2x - 20)$.

Lösung A3

a) $f'(x) = -2 \left(x^2 \sin \left(\frac{1}{2}x^2 + 4 \right) - \cos \left(\frac{1}{2}x^2 + 4 \right) \right)$

$$f''(x) = -2 \left(3x \sin \left(\frac{1}{2}x^2 + 4 \right) + x^3 \cos \left(\frac{1}{2}x^2 + 4 \right) \right)$$

b) $f'(x) = x^2 \cdot (3 \sin(x) + x \cos(x))$

$$f''(x) = -x((x^2 - 6)\sin(x) - 6x\cos(x))$$

c) $f'(x) = 2(\cos^2(x) - \sin^2(x))$

$$f''(x) = -8 \cos(x) \sin(x)$$

d) $f'(x) = 3(x^2 - \sin(x))^2 \cdot (2x - \cos(x))$

$$f''(x) = 3(x^2 - \sin(x))^2(\sin(x) + 2) + 6(2x - \cos(x))^2(x^2 - \sin(x))$$

e) $f'(x) = 2a(\cos(ax) - 1) \cdot (\sin(ax) - ax)$

$$f''(x) = -2a^2(\sin^2(ax) - ax\sin(ax) - \cos^2(ax) + 2\cos(ax) - 1)$$

f) $f'(x) = -x\sin^2(x) + \sin(x)\cos(x) + x\cos^2(x)$

$$f''(x) = -2(\sin^2(x) + 2x\sin(x)\cos(x) - \cos^2(x))$$

g) $f'(x) = 2x(\sin(4x+3) + 2x \cdot \cos(4x+3))$

$$f''(x) = (2 - 16x^2)\sin(4x+3) + 16x\cos(4x+3)$$