Lösung A1

a) $f(x) = ln(x) + 4x^2 + 3$

$f'(x) = \frac{1}{x} + 8x$

b) $f(x) = (2x-1)(5x+3)$

$= 10x^2 + x - 3$

$f'(x) = 20x + 1$

c) $f(x) = 2x^2 ln(x)$

Produktregel erforderlich

$u = 2x^2; \quad u' = 4x$

$v = \ln(x); \quad v' = \frac{1}{x}$

$f'(x) = u'v + v'u = 4xln(x) + \frac{2x^2}{x} = 4xln(x) + 2x$

d) $f(x) = ln\left(\frac{x+1}{x-1}\right)^2 = 2 \cdot ln\left(\frac{x+1}{x-1}\right)$

Kettenregel und Quotientenregel erforderlich

$u = 2 \cdot ln(v); \quad u' = \frac{2}{v} \cdot v'$

$v = \frac{x+1}{x-1} = \frac{w}{q}; \quad v' = \frac{w'q - q'w}{q^2}$

$w = x + 1; \quad w' = 1$

$q = x - 1; \quad q' = 1 \;\; ; \;\; q^2 = (x-1)^2$

$v' = \frac{1\cdot(x-1) - 1\cdot(x+1)}{(x-1)^2} = \frac{-2}{(x-1)^2}$

$f'(x) = \frac{2}{\frac{x+1}{x-1}} \cdot \frac{-2}{(x-1)^2} = -\frac{4\cdot(x-1)}{(x+1)\cdot(x-1)^2} = -\frac{4}{(x+1)(x-1)} = -\frac{4}{x^2-1}$

e) $f(x) = ln(x^3 - 4x^2)$

Kettenregel erforderlich

$f'(x) = \frac{1}{x^3 - 4x^2} \cdot (3x^2 - 8x)$

f) $f(x) = \frac{2x^3 + x}{e^x}$

Quotientenregel erforderlich

$u = 2x^3 + x; \quad u' = 6x + 1$

$v = e^x; \quad v' = e^x; \quad v^2 = e^{2x}$

$f'(x) = \frac{(6x+1)e^x - (2x^3+x)e^x}{e^{2x}}$

$f'(x) = \frac{e^x(6x+1-2x^3-x)}{e^{2x}} = \frac{-2x^3+5x+1}{e^x}$

g) $f(x) = ln(x+3)^2$

Kettenregelerforderlich

$f'(x) = 2ln(x+3) \cdot \frac{1}{x+3}$

$f'(x) = \frac{2ln(x+3)}{x+3}$

h) $f(x) = e^{\sqrt{x^2+5}} = e^{\frac{1}{2}(x^2+5)} = e^{\frac{x^2}{2}+\frac{5}{2}}$

Kettenregel erforderlich

$f'(x) = x \cdot e^{\sqrt{x^2+5}}$

i) $f(x) = \frac{x \cdot ln(x)}{x+1}$

Produktregel und Quotientenregel erforderlich

$u = x \cdot ln(x); \quad u' = \ln(x) + x \cdot \frac{1}{x} = \ln(x) + 1$

$v = x + 1; \quad v' = 1; \quad v^2 = (x+1)^2$

$f'(x) = \frac{u'v - v'u}{v^2} = \frac{(ln(x)+1)\cdot(x+1) - x\cdot ln(x)}{(x+1)^2} = \frac{xln(x) + ln(x) + x + 1 - xln(x)}{(x+1)^2}$

$= \frac{ln(x) + x + 1}{(x+1)^2}$

j) $f(x) = \sqrt{x-1} = (x-1)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2} \cdot (x-1)^{-\frac{1}{2}} = \frac{1}{2 \cdot \sqrt{x-1}}$

k) $f(x) = (x^3+2)(x+3) = x^4 + 3x^3 + 2x + 6$

$f'(x) = 4x^3 + 9x^2 + 2$

l) $f(x) = \frac{1}{\sqrt[3]{x^3+1}} = (x^3+1)^{\frac{1}{3}}$

$f'(x) = \frac{1}{3} \cdot (x^3+1)^{-\frac{2}{3}} \cdot 3x^2 = \frac{x^2}{\sqrt[3]{(x^3+1)^2}}$

Lösung A2

a) $f(x) = 5e^{2x^2-3x+1}$

Kettenregel erforderlich

$f'(x) = 5 \cdot (6x-3) \cdot e^{2x^2-3x+1}$

b) $f(x) = \frac{x^3}{ln(x)}$

Quotientenregel erforderlich

$u = x^3; \quad u' = 3x^2$

$v = ln(x); \quad v' = \frac{1}{x}; \quad v^2 = ln^2(x)$

$f'(x) = \frac{u'v - v'u}{v^2} = \frac{3x^2 \cdot ln(x) - \frac{x^3}{x}}{ln^2(x)} = \frac{x^2(3\,ln(x)-1)}{ln^2(x)}$

c) $f(x) = \sqrt[3]{x^2 + ln(x^2)} = (x^2 + \ln(x^2))^{\frac{1}{3}}$

Kettenregel erforderlich

$f'(x) = \frac{1}{3} \cdot (x^2 + \ln(x^2))^{-\frac{2}{3}} \cdot (2x + \frac{1}{x^2} \cdot 2x)$

$f'^{(x)} = \frac{2x + \frac{2}{x}}{3 \cdot \sqrt[3]{(x^2+\ln(x^2))^2}} = \frac{2x^2+2}{3x \cdot \sqrt[3]{(x^2+\ln(x^2))^2}} = \frac{2(x^2+1)}{3x \cdot \sqrt[3]{(x^2+\ln(x^2))^2}}$

d) $f(x) = \frac{e^x}{ln(x+1)}$

Quotientenregel erforderlich

$u = e^x; \quad u' = e^x$

$v = ln(x+1); \quad v' = \frac{1}{x}; \quad v^2 = ln^2(x+1)$

$f'(x) = \frac{u'v - v'u}{v^2} = \frac{e^x \cdot ln(x+1) - e^x \frac{1}{x}}{ln^2(x+)} = \frac{e^x\left(ln(x+1) - \frac{1}{x}\right)}{ln^2(x+1)}$