Lösung A1

a) $f'(x) = \frac{3x(x\sin(x)+2\cos(x))}{\cos^2(x)}$

$f''(x) = \frac{6x^2\sin^2(x)+12x\sin(x)\cos(x)+(3x^2+6)\cos^2(x)}{\cos^3(x)}$

b) $f'(x) = \frac{(2x+1)\sin(x)+2\cos(x)}{3\cdot\sqrt[3]{(2x+1)^2\cdot\cos^4(x)}}$

$f''(x) = \frac{(16x^2+16x+4)\sin^2(x)+(8x+4)\sin(x)\cos(x)+(12x^2+12x-5)\cos^2(x)}{9\cdot\sqrt[3]{(2x+1)^5\cdot\cos^7(x)}}$

c) $f'(x) = \frac{x\sin^2(x)(2\sin(x)+3x\cos(x))}{2}$

$f''(x) = \frac{\sin(x)\left((3x^2-2)\sin^2(x)-12x\sin(x)-6x^2\cos^2(x)\right)}{2}$

d) $f'(x) = \frac{(x\sin(x)-2\cos(x))\sin(x^2-1)+2x^2\cos(x)\cos(x^2-1)}{2x\cos^2(x)\cdot|x|\cdot\sqrt{\frac{\sin(x^2-1)}{\cos x}}}$

$f''(x) = \frac{(3x^2\sin^2(x)-4\sin(x)\cos(x)+(-8x^4+2x^2+8)\cos^2(x))\sin^2(x^2-1)}{4x^2\cos^4(x)\cdot|x|\cdot\sqrt{\left(\frac{\sin(x^2-1)}{\cos x}\right)^3}} +$

$\frac{(4x^3\sin(x)\cos(x)-4x^2\cos^2(x))\cos(x^2-1)\sin(x^2-1)-4x^4\cos^2(x)\cos^2(x^2-1)}{4x^2\cos^4(x)\cdot|x|\cdot\sqrt{\left(\frac{\sin(x^2-1)}{\cos x}\right)^3}}$

e) $f'(x) = \frac{x(x\sin(x)+2\cos(x)+4)}{(2+\cos(x))^2}$

$f''(x) = \frac{2x^2\sin^2(x)+\sin(x)(4x\cos(x)+8x)+\cos^2(x)(x^2+2)+\cos(x)(2x^2+8)+8}{(2+\cos(x))^3}$

f) $f'(x) = (2-2)\cdot\ln(x^2+1) + \frac{2x(x^2-2x)}{x^2+1}$

$f''(x) = \frac{2((x^4+2x^2+1)\cdot\ln(x^2+1)+3x^4-2x^3+5x^2-6x)}{(x^2+1)^2}$

g) $f'(x) = \frac{2e^{4x}\cdot\cos(\sqrt{e^{4x}+5})}{\sqrt{e^{4x}+5}\cdot\sin(\sqrt{e^{4x}+5})}$

$f''(x) =$

$\frac{4e^{4x}\left(e^{4x}\cdot(e^{4x}+5)^{\frac{3}{2}}\cdot\sin^2(\sqrt{e^{4x}+5})+(-e^{8x}-15e^{4x}-50)\cdot\cos(\sqrt{e^{4x}+5})\cdot\sin(\sqrt{e^{4x}+5})+e^{4x}(e^{4x}+5)^{\frac{3}{2}}\cdot\cos^2(\sqrt{e^{4x}+5})\right)}{(e^{4x}+5)^{\frac{5}{2}}\cdot\sin^2(\sqrt{e^{4x}+5})}$

h) $f(x) = \frac{\tan(x)}{x} = \frac{\sin(x)}{x\cdot\cos(x)}$ $\qquad f'(x) = \frac{\sin^2(x)}{x\cdot\cos^2(x)} - \frac{\sin(x)}{x^2\cos(x)} + \frac{1}{x}$

$f''(x) = \frac{2(x^2\sin^3(x)-x\cos(x)\sin^2(x)+(x^2+1)\cos^2(x)\sin(x)-x\cos^3(x))}{x^3\cos^3(x)}$

Lösung A2

a) $f'(x) = 6x - \frac{1}{\sqrt{x}} + \frac{1}{3x^{\frac{4}{3}}}$ $\qquad f''(x) = \frac{1}{2x^{\frac{3}{2}}} - \frac{5}{9x^{\frac{7}{3}}} + 6$

$f'''(x) = \frac{28}{27x^{\frac{10}{3}}} - \frac{3}{4x^{\frac{5}{2}}}$

b) $f'(x) = 36x^3 - 8x + 4$ $\qquad f''(x) = 108x^2 - 8$

$f'''(x) = 216x$

c) $f'(x) = 5\cos(x) - 3\sin(x)$ $\qquad f''(x) = -5\sin(x) - 3\cos(x)$

$f'''(x) = -5\cos(x) + 3\sin(x)$

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Lösung A3

$f(x) = 1 + tan^2(x) = 1 + \frac{sin^2(x)}{cos^2(x)}$ $\quad u = sin^2(x) \quad u' = 2\,sin(x)\,cos(x)$

$v = cos^2(x) \quad u' = -2sin(x)cos(x)$

$$f'(x) = \frac{2\,sin(x)\,cos^3(x) + 2\,sin^3(x)\,cos(x)}{cos^4(x)} = \frac{2\,sin(x)\,cos^2\,x + 2\,sin^3(x)}{cos^3(x)}$$

$$= \frac{2sin(x)(cos^2\,x + sin^2(x))}{cos^3(x)} = 2 \cdot \frac{sin(x)}{cos(x)} \cdot \frac{1}{cos^2(x)} = 2tan(x) \cdot sec^2(x)$$

q.e.d.

$f'(x) = 2tan(x) \cdot sec^2(x)$ $\quad u = 2tan(x) \quad u' = 2\,sec^2(x)$

$v = sec^2(x) \quad u' = 2\,sec^2(x)\,tan(x)$

$f''(x) = 2\,sec^4(x) + 4\,sec^2(x) \cdot tan^2(x)$ **q.e.d.**

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