Lösung A1

a) $\sqrt{\frac{4a^2}{9b^2}} = \frac{2a}{3b}$

b) $\sqrt{\frac{16x^6}{81y^4}} = \frac{4x^3}{9y^2}$

c) $\sqrt{x^2-1} \cdot \sqrt{\frac{x+1}{x-1}} = \sqrt{(x+1)(x-1) \cdot \frac{x+1}{x-1}} = \sqrt{(x+1)^2} = x+1$

d) $(x-1) \cdot \sqrt[3]{\frac{x^2+2x+1}{x^2-1}} = \sqrt[3]{(x-1)^3 \cdot \frac{(x+1)^2}{(x+1)(x-1)}} = \sqrt[3]{(x-1)^2 \cdot (x+1)}$

e) $\left(\frac{x-1}{x+1}\right) \cdot \sqrt[3]{\frac{(x^2+2x+1)^2}{(x^2-1)^2}} = \sqrt[3]{\frac{(x-1)^3 \cdot (x+1)^4}{(x+1)^3 \cdot (x+1)^2 \cdot (x-1)^2}} = \sqrt[3]{\frac{(x-1)^3 \cdot (x+1)^4}{(x-1)^2 \cdot (x+1)^5}} = \sqrt[3]{\frac{x-1}{x+1}}$

f) $\sqrt{a^2-b^2} \cdot \sqrt{\frac{5a+5b}{a-b}} = \sqrt{(a+b)(a-b) \cdot \frac{5(a+b)}{a-b}} = \sqrt{5(a+b)^2} = (a+b) \cdot \sqrt{5}$

g) $\sqrt{5a^2-5b^2} \cdot \sqrt{\frac{5a+5b}{a-b}} = \sqrt{5(a+b)(a-b) \cdot \frac{5(a+b)}{a-b}} = 5(a+b)$

h) $(x+y) \cdot \sqrt{\frac{x^2-y^2}{(x^2+2xy+y^2)(x+y)}} = \sqrt{\frac{(x+y)^2 \cdot (x+1)(x-1)}{(x+y)^2 \cdot (x+y)}} = \sqrt{x-1}$

i) $0{,}5e\sqrt{e^{-2}} + 2e = 0{,}5e \cdot \sqrt{\frac{1}{e^2}} + 2e = 0{,}5 + 2e$

Lösung A2

a) $\sqrt[3]{x^2+2xy+y^2} : \sqrt[3]{x^2-y^2} = \sqrt[3]{\frac{(x+y)^2}{(x+y)(x-y)}} = \sqrt[3]{\frac{x+y}{x-y}}$

b) $(x+y) \cdot \sqrt{\frac{x^2+y^2}{(x^2+2xy+y^2)}} = \sqrt{(x+y)^2 \cdot \frac{x^2+y^2}{(x+y)^2}} = \sqrt{x^2+y^2}$

c) $\sqrt[n]{\frac{b^{-3}c^{-3}a^{2n}}{b^{-3n}dc^{-5}a^{-2}} : \frac{a^2c^2\,b^n}{a^{-n}da^nb^3}} = \sqrt[n]{\frac{b^{-3}c^{-3}a^{2n}}{b^{-3n}dc^{-5}a^{-2}} \cdot \frac{a^{-n}da^nb^3}{a^2c^2\,b^n}} = \sqrt[n]{\frac{a^{2n}b^0c^{-3}d}{a^0b^{-2n}c^{-3}d}} = \sqrt[n]{(ab)^{2n}} = (ab)^2$

d) $\sqrt[a]{\frac{y^{3b}y^{-2}x^{2a}z^3}{y^{-2}x^{-3b}z^{-a}} : \frac{z^3\,x^{3b}}{y^{-3b}x^{-2a}}} = \sqrt[a]{\frac{y^{3b}y^{-2}x^{2a}z^3}{y^{-2}x^{-3b}z^{-a}} \cdot \frac{y^{-3b}x^{-2a}}{z^3\,x^{3b}}} = \sqrt[a]{\frac{y^{-2}z^3}{y^{-2}z^3z^{-a}}} = \sqrt[a]{z^a} = z$

e) $\sqrt{8k^2-16k+8} = \sqrt{8(k^2-2k+1)} = \sqrt{4 \cdot 2 \cdot (k-1)^2} = 2(k-1) \cdot \sqrt{2}$

f) $\left(1+\sqrt{k}\right)^2 = 1 + 2\sqrt{k} + k$

g) $\left(\sqrt{a}-2\sqrt{b}\right)^2 = a - 4\sqrt{ab} + 4b$

h) $\sqrt{0{,}25k} - \sqrt{\frac{k}{25}} + 3\sqrt{k} = 0{,}5\sqrt{k} - 0{,}2\sqrt{k} + 3\sqrt{k} = 3{,}3\sqrt{k}$

i) $\left(\sqrt{x}-\sqrt{2}\right)\left(\sqrt{x}+\sqrt{2}\right) = x-2$

j) $\sqrt{xy^2} - 5\sqrt{x^2y} + 8x\sqrt{y} - 10y\sqrt{x} = 3x\sqrt{y} - 9\sqrt{x}$

Lösung A3

a) $\frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}} = \frac{2}{5}\sqrt{5}$

b) $\frac{x}{2\sqrt{x}} = \frac{x\sqrt{x}}{2\sqrt{x}\sqrt{x}} = \frac{1}{2}\sqrt{x}$

c) $\frac{1+\sqrt{k}}{1-\sqrt{k}} = \frac{(1+\sqrt{k})(1+\sqrt{k})}{(1-\sqrt{k})(1+\sqrt{k})} = \frac{(1+\sqrt{k})^2}{1-k}$

d) $\frac{k}{\sqrt{5k}-\sqrt{3k}} = \frac{k(\sqrt{5k}+\sqrt{3k})}{(\sqrt{5k}-\sqrt{3k})(\sqrt{5k}+\sqrt{3k})} = \frac{k(\sqrt{5k}+\sqrt{3k})}{5k-3k} = \frac{1}{2}(\sqrt{5k}+\sqrt{3k})$

e) $\frac{\sqrt{x-1}}{\sqrt{x}-1} = \frac{\sqrt{x-1}\cdot(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = \frac{\sqrt{x-1}\cdot(\sqrt{x}+1)}{x-1}$

f) $\sqrt{a}+1+a-\frac{a}{\sqrt{a}} = \sqrt{a}+1+a-\frac{a\cdot\sqrt{a}}{\sqrt{a}\cdot\sqrt{a}} = 1+a$

Lösung A4

a) $\frac{1}{9k}(\sqrt{k})^5 + \frac{1}{9}(\sqrt{k})^3 + \frac{3}{2}k\sqrt{k} = \frac{1}{9}k\sqrt{k} + \frac{1}{9}k\sqrt{k} + \frac{3}{2}k\sqrt{k} = \frac{31}{18}k\sqrt{k}$

b) $-\frac{1}{2k}\left((-\sqrt{k})^4 + k(-\sqrt{k})^2\right) = -\frac{1}{2}k - \frac{1}{2}k = -k$

c) $-\frac{k^2}{144}\cdot\left(\frac{6}{\sqrt{k}}\right)^3 + \frac{k}{2}\cdot\left(\frac{6}{\sqrt{k}}\right) = -\frac{216k}{144\sqrt{k}} + \frac{3k}{\sqrt{k}} = -\frac{3k}{2\sqrt{k}} + \frac{3k}{\sqrt{k}} = \frac{3k\cdot\sqrt{k}}{2\sqrt{k}\cdot\sqrt{k}} = \frac{3}{2}\sqrt{k}$

d) $\frac{1}{k^2}\left(\sqrt{0{,}5k}\right)^3 - \frac{3}{2k}\left(\sqrt{0{,}5k}\right)^2 + 2 = \frac{0{,}5}{k}\sqrt{0{,}5k} - \frac{1{,}5}{2} + 2 = \frac{\sqrt{2k}}{4k} + \frac{5}{4}$

Lösung A5

a) $\sqrt{8} - 3\sqrt{2} = \sqrt{4\cdot 2} - 3\sqrt{2} = 2\sqrt{2} - 3\sqrt{2} = -\sqrt{2}$

b) $\sqrt{18} - 3\sqrt{8} = \sqrt{9\cdot 2} - 3\sqrt{4\cdot 2} = 3\sqrt{2} - 6\sqrt{2} = -3\sqrt{2}$

c) $\sqrt{5} + \sqrt{20} - \sqrt{25} = \sqrt{5} + \sqrt{4\cdot 5} - 5 = 3\sqrt{5} - 5$

d) $(\sqrt{5})^3 + 3\sqrt{5^2} - (4\sqrt{5})^2 + \sqrt{5}\sqrt{5^3} = 5\sqrt{5} + 15 - 80 + 25 = 5\sqrt{5} - 40$

e) $\sqrt{3^4} + (-2\sqrt{3})^4 + (\sqrt{3^2})^3 - 5\sqrt{3}\sqrt{3^3} = 9 + 16\cdot 9 + 3\cdot 9 - 5\cdot 9 = 15\cdot 9 = 135$